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Name	
Roll No	
Program	
Course Code	DCA1203
Course Name	Mathematics
Semester	1^{st}

Question .1) State inclusion-exclusion principle. In a class of 1000 students, 625 students pass in Mathematics and 525 pass in Data Structure. How many students pass in Mathematics only and how many students pass in Data Structure only?

Solution .:-

Inclusion-Exclusion Principle

In a class of 1000 students:

- 625 students pass in Mathematics
- 525 students pass in Data Structure

Let **M** denote the set of students passing in Mathematics (625 students) and **D** denote the set of students passing in Data Structure (525 students).

The inclusion-exclusion principle formula is given as:

Total = $|M| + |D| - |M \cap D|$

Where:

- $|M|$ represents the number of students passing in Mathematics.
- $|D|$ represents the number of students passing in Data Structure.
- $|M \cap D|$ represents the number of students passing in both Mathematics and Data Structure.

Given:

- $|M| = 625$ (students passing in Mathematics)
- $|D| = 525$ (students passing in Data Structure)

Using the formula:

$|M \cap D| = |M| + |D| - \text{Total}$

Calculating $|M \cap D|$:

$|M \cap D| = 625 + 525 - 1000$

$|M \cap D| = 1150 - 1000$

$|M \cap D| = 150$

Therefore, 150 students pass in both Mathematics and Data Structure.

Now, to find the number of students passing in Mathematics only:

Mathematics only = $|M| - |M \cap D|$

Mathematics only = 625 - 150
Mathematics only = 475

And for the number of students passing in Data Structure only:
Data Structure only = |D| - |M∩D|
Data Structure only = 525 - 150
Data Structure only = 375

Therefore, in summary:
Mathematics only:.- 475 students
Data Structure only:.- 375 students

Question.2) Simplify2 $= \frac{(\cos 0 + i\sin 0)^5}{(\cos 0 - i\sin 0)^4}$ ***into*** $x +$ ***iy from and find its modulus and the amplitude .***

Solution .:-

This expression involves De Moriver's theorem ,which states that *(cos 0 + I sin 0)*^{2} *= cos(n0) + i sin (n0) for any integer n .*

Applying De Moriver's theorem

Numerator : (cos 0 + i sin0)^5 = cos(50) + i sin (50)

Denominator **:** (cos 0 + i sin0)^4 = cos(40) + i sin (40)

So , the simplified expression becomes :

$$\frac{\cos(50) + i\sin(50)}{\cos(40) - i\sin(40)}$$

Now , to write this in the form of x + iy , let's rationalize the denominator by multiplying both the numerator and denominator by the conjugate of the denominator :

$$\frac{(\cos(50) + i\sin(50))(\cos(40) - i\sin(40))}{\cos^2(40) - i\sin^2(40)}$$

This simplifies to :

$$\frac{\cos(50)\cos(40) + \sin(50)\sin(40) + i(\sin(50)\cos(40) - \cos(50)\sin(40))}{1}$$

Cos(50) Cos(40) + sin(50) sin(40) + i (sin (50) cos (40) – cos(50) sin(40))

This can be further simplified using trigonometric identities:

Cos(50) Cos(40) + sin(50) sin(40) + i (sin (50) cos (40) – cos(50) sin(40))

cos (50 – 40) + i sin (50 – 40)

cos (0) + i sin (0)

So , the expression simplifies to cos (0) + i sin (0) , which represents a complex number in polar form , equivalent to e^{i0} .

The modulus (or absolute values) of this complex number is 1(since $\cos^2(0) + \sin^2(0) = 1$ in trigonometry) , and the amplitude is the angle , which is 0 .

Question.3.a.) Evaluate $\int \log x\, dx$.

Solution .:-

To evaluate the integral $\int \log x\, dx$, we can use integration by parts

Let u = log x and dv = dx. Then, by differentiating and integrating:-

$du = 1/x\, dx$

v = x

Using the formula for integration by parts:-

$\int u\, dv = uv - \int v\, du$

we can substitute our values:-

$\int \log x\, dx = x \log x -$

$\int x.1/x\, dx$ Simplifying

$\int \log x\, dx = x \log x - \int dx$

The integral $\int dx$ is simply x,

$\int \log x\, dx = x \log x - x + C$

Where C is the constant of integration.

Question.3.b.)

Solve the equation (2x-y+1 dx + (2y – x – 1) dy = 0 .

Solution .:-

Step 1: Check for exactness

First, we need to check if the equation is exact. To do this, we calculate the partial derivatives of M(x, y) = 2x - y + 1 and N(x, y) = 2y - x - 1:

- $\partial M/\partial y = -1$

- $\partial N/\partial x = -1$

Notice that $\partial M/\partial y = \partial N/\partial x$. This is a necessary condition for the equation to be exact.

Step 2: Find the potential function

Since the condition is met, we can find a potential function u(x, y) such that:

- $\partial u/\partial x = M(x, y) = 2x - y + 1$
- $\partial u/\partial y = N(x, y) = 2y - x - 1$

Integrating the first equation with respect to x:

- $u(x, y) = x^2 - xy + x + v(y)$

where v(y) is an arbitrary function of y only. Now, taking the partial derivative of this u with respect to y:

- $\partial u/\partial y = -x + v'(y)$

Setting this equal to the second equation for N(x, y):

- $2y - x - 1 = -x + v'(y)$

Solving for v'(y):

- $v'(y) = 2y + 1$

Integrating this with respect to y:

- $v(y) = y^2 + y + C$

where C is another arbitrary constant.

Step 3: Write the general solution

Substituting v(y) back into u(x, y):

- $u(x, y) = x^2 - xy + x + y^2 + y + C$

Therefore, the general solution of the given differential equation is:

$x^2 - xy + x + y^2 + y + C = 0$

where C is an arbitrary constant.

This gives us a relationship between x and y. However, it's important to remember that this is just one possible solution out of a family of solutions parameterized by the constant C.

Question.4.a.) Evaluate the followings :

i) $\lim\limits_{n \to \infty} \frac{2+n+n^2}{2+3n+4n^2}$

Solution .:-

To find the limit of the expression $\frac{2+n+n^2}{2+3n+4n^2}$ as n approaches infinity, we can divide both the numerator and denominator by the highest power of n *in the expression.*

$$\lim_{n \to \infty} \frac{2+n+n^2}{2+3\,n+4n^2} = \lim_{n \to \infty} \frac{n^2\,(\frac{2}{n^2} + \frac{2}{n^2} + 1)}{n^2\,(\frac{2}{n^2} + \frac{3}{n} + 4)}$$

Simplifying further:

$$\lim_{n \to \infty} \frac{n^2\,(\frac{2}{n^2} + \frac{2}{n^2} + 1)}{n^2\,(\frac{2}{n^2} + \frac{3}{n} + 4)} = \lim_{n \to \infty} \frac{2+\frac{1}{n}+\frac{1}{n^2}}{2+\frac{3}{n}+\frac{4}{n^2}}$$

As n approaches infinity, terms involving $\frac{1}{n}$ *or* $\frac{1}{n^2}$ become negligible compared to constants, so:

$$\lim_{n \to \infty} \frac{2+\frac{1}{n}+\frac{1}{n^2}}{2+\frac{3}{n}+\frac{4}{n^2}} = \frac{2}{2} = 1$$

Therefore, the limit of the expression as n approaches infinity is 1.

ii) $\lim\limits_{x \to 2} \frac{2x^2-3x-2}{x-2}$

Solution .:-

As x approaches 2, let's directly substitute $x = 2$ into the expression:

$$\frac{2x^2-3x-2}{x-2} = \frac{2(2)^2-3(2)-2}{2-2}$$

Solving this now,

$$\frac{2(2)-6-6}{0} = \frac{8-6-2}{0} = \frac{0}{0}$$

The result is an indeterminate form of $\frac{0}{0}$, which means we can't determine the limit immediately by direct substitution.

To resolve this, let's factorize the numerator:

$2x^2 - 3x - 2 = (2x + 1)(x - 2)$

Now, let's simplify the expression again:

$$\frac{2x^2-3x-2}{x-2} = \frac{(2x+1)(x-2)}{x-2}$$

Canceling out the common factor of x 2, we get .:

$$\frac{(2x+1)(x-2)}{x-2} = 2x+1$$

Now, if we substitute x 2 into x +1 :

Question.4.b) **Find the probability of drawing a diamond card in each of the two consecutive draw from a pack of well shuffled 52 cards , (i) if the card is replaced , (ii) if the following is not replaced .**

Solution .:-

Case (i): Card is replaced

In this case, the probability remains the same for both draws, as the deck is effectively "reset" after each draw. Here's how to calculate the probability:

1. Probability of drawing a diamond on the first draw: There are 13 diamonds in the deck and 52 total cards, so the probability is 13/52.
2. Probability of drawing a diamond on the second draw (card replaced): Again, there are still 13 diamonds and 52 cards, so the probability remains 13/52.

Therefore, the overall probability of drawing a diamond card in both draws with replacement is:

Probability = (13/52) * (13/52) = 0.0625

Case (ii): Card is not replaced

Here, the probability changes for the second draw because there are one fewer diamond and one fewer card in the deck after the first draw.

1. Probability of drawing a diamond on the first draw: Same as before, 13/52.
2. Probability of drawing a diamond on the second draw (card not replaced): After the first draw, there are only 12 diamonds left and 51 cards remaining. So, the probability is 12/51.

Therefore, the overall probability of drawing a diamond card in both draws without replacement is:

Probability = (13/52) * (12/51) ≈ 0.0588

So, as you can see, the probability is slightly lower when the card is not replaced due to the reduced number of diamonds available in the second draw.

Question.5.)

Check whether the following is Tautology or Contradiction:

i) $(p \vee q) \vee (\sim p)$

ii) $\sim [p \vee (\sim p)]$

Solution .:-

i) $(p \vee q) \vee (\sim p)$

p	**q**	**~p**	**p V q**	**(p V q) V (q)**
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

From the truth table , *(q V q) V (~p)* is always true, regardless of the values of *p* and *q* *So, it's a tautology.*

ii) $\sim [p \vee (\sim p)]$

p	**~p**	**P V (~p)**	**~[p V (~p)]**
T	F	T	F
T	T	T	F

The truth table shows that ~[p V (~p)] is always false, regardless of the value of *p* .

Hence, it's a contradiction.

The truth tables confirm that the first expression is a tautology, and the second one is a contradiction.

Question.6.**)**
Apply Cramer's rule to solve the system of equations:

$$3x + y + 2z = 3; \quad 2x - 3y - z = -3; \quad x + 2y + z = 4.$$

Solution .:- let's denote the coefficients of the variables

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ 2 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$$

According to Cramer's rule, the value of each variable is given by the determinants of matrices formed by replacing each column of A with the column matrix B, divided by the determinant of A.

Now, let's compute the determinant of A :

$$|A| = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

Applying cofactor expansion along the first row, we have:-

$$|A| = 3\begin{vmatrix} -3 & -1 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix}$$

Evaluating the 2x2 determinants:

$|A| = 3 ((-3) (1) - (-1) (2)) – 1 ((2) (1) – (-1) (1) + 2 ((2) (2) – (-3) (1))$

$|A| = 3 (-5) – 1 (3) + 2 (7) = - 15 – 3 + 14 = - 4$

The determinant of matrix A is - 4.

Next, we will compute the determinant of matrices obtained by replacing each column of A with B :

For the determinant of matrix A , we replace the first column of A with the column matrix B :

$$Ay = \begin{bmatrix} 3 & 3 & 2 \\ 2 & -3 & -1 \\ 1 & 4 & 1 \end{bmatrix}$$

Applying cofactor expansion along the second column, we have:

$$Ay = 3\begin{vmatrix} 3 & 2 \\ 2 & 2 \end{vmatrix} - (3)\begin{vmatrix} 3 & 3 \\ 1 & 1 \end{vmatrix} + 2\begin{vmatrix} 3 & 3 \\ 1 & 4 \end{vmatrix}$$

Evaluating the 2x2 determinants:

$Ay = 3 ((3) (-1) - (2) (-1)) – (-3)((3)(1) – (2) (1)) + 2 ((3) (4) – (3) (1))$

$Ay = 3(−1)−(−3)(1)+2(9)=−3+3+18=$

The determinant of matrix Ay is 18.
Finally, for the determinant of matrix Az , we replace the third column of A with the column matrix B.

$$Az = \begin{bmatrix} 3 & 1 & 3 \\ 2 & -3 & 4 \\ 1 & 2 & 2 \end{bmatrix}$$

Applying cofactor expansion along the third column, we have:

$$Az = 3\begin{vmatrix} 3 & -3 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix}$$

Evaluating the 2x2 determinants:

Az= 3 ((2) (2) - (-3) (1)) – 1((2)(4) – (-3) (1)) + 3 ((2) (2) – (-3) (2))

Az=3(4+3)−1(8+3)+3(4+6)=21−11+30=40

The determinant of matrix Az is 40.

Finally, we can find the values of the variables using Cramer's rule:

$$\boldsymbol{x = \frac{Ax}{|A|} = \frac{-12}{-4} = 3 \quad y = \frac{Ay}{|A|} = \frac{-18}{-4} = \frac{9}{2} \quad z = \frac{Az}{|A|} = \frac{40}{-4} = -10}$$

Therefore, the solution to the system of equations is $x = 3$,$y = -\frac{9}{2}$ $and\ z = -10$